Combination resonance in a multi-excited weakly nonlinear vibration absorber

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Abstract

It is established that oscillators with a weakly nonlinear spring, also resonates for excitations with a different frequency than its natural frequency. The studied type of resonance is called combination resonance, that occurs if the natural frequency approximately equals a sum of the excitation frequencies. This resonance can dissipate energy from several frequencies simultaneously. If a weakly nonlinear oscillator is used as an absorber on a main system, it can damps several vibration modes simultaneously. If combination resonance occurs in the absorber, it vibrates with the excited frequencies and its own natural frequency, which is higher as it is a sum of the excitation frequencies. This implies a potentially higher speed and dissipation. A tuning method is proposed to ensure this resonance in the weakly nonlinear absorbers. Additionally, a structural modification is added to the absorber, which increases the range of vibration energy of the main system that yields combination resonances. Simulations are performed that validate the theoretical analysis and tuning.

1 Introduction

In passive vibration absorption, excessive vibrations in a *main system* are mitigated by adding one (or several) passive absorbing element(s), called a vibration absorber. The main system is the structure that needs protection from vibrations; a bridge, a building, a metal structure, an airplane wing, car bodywork, ... A passive element is a correctly dimensioned structure, which when added to the main system, absorbs or counteracts vibrations in the main system. As opposed to active absorption, passive absorption requires no power source or sensor, making it an elegant and robust solution to vibration reduction in engineering structures.

As early as 1940, Den Hartog [1] devised an analytic method to optimally tune the mass, spring and viscous damping parameters of a single vibration absorber for single-degree-of-freedom (SDOF) main systems. Vibrations in the main system are succesfully attenuated, but only if the main system is excited near its resonant frequency. Moreover, the compound system now has two resonant frequencies. If the main system is then excited near one these new resonant frequencies, excessive vibration remains possible.

In search for absorbers operable over a larger excitation bandwidth, and capable of transient vibration absorption, Gendelman discovered the targeted energy transfer (TET) for transient vibrations [2] for absorber with a strong nonlinear stiffness. The TET phenomenon is the sudden transfer of vibration energy from the main system to the absorber, where the energy is then dissipated. TET does not depend on the vibration frequency of the main system, but on the initial conditions of the main system. This vastly increases the operable frequency bandwidth of the absorber. Vakakis et al.[3] expanded this work to MDOF main systems that vibrate with multiple frequencies simultaneously. TET then occurs for each frequency sequentially, from highest to lowest frequency. Further research reformulated the dependence of initial conditions to the more general initial energy and applied TET to harmonic forcing of the main system [4, 5]. Existing research on these multi-frequency vibrating systems assume that the absorber vibrates with only one (possibly changing in time) frequency, or at best, two frequencies where one frequency is an integer multiple of the lower [6].

Nayfeh [7, 8] showed that SDOF systems with a weakly nonlinear spring exhibits the so called *combination resonance*. This occurs if this system is excited with several frequencies, and when a linear combination of these frequencies approximately equals the natural frequency of the system. The nonlinear system then vibrates persistently with both the excitation frequencies and its natural frequency. This is in contrast to purely linear systems, where the natural frequency term is transient. In this study it is shown that with the imposed motion method, proposed in [9], the vibrations of a main system can be seen as excitations on the added absorber. These excitations can impose combination resonance in the added absorber. As with TET, it was found that the energy in the main system should be above some threshold for combination resonance to occur in the absorber. To lower this threshold, a configuration is proposed where the absorber is additionally connected to the ground with a negative stiffness. A negative stiffness can be constructed by an axially loaded beam between two buckling states, and has been proven to reduce vibrations in main systems [10]. A tuning method is proposed, using the so called *combination FRF*. To start tuning, initial conditions in the main system and the linear part of the absorber should be known. Then, by adjusting the damping and small nonlinearities, the working point of the absorber in this combination FRF is shifted to ensure combination resonance. It is important to distinguish between what this research demonstrates and the sequential frequency capture of TET [3]. In this study, as long as all frequencies are present, combination resonance will persist and absorb energy from all frequencies participating in the so-called *combination*. With sequential absorption, only 1 frequency at a time is absorbed. Another reason to study combination resonances is that the natural frequency is higher than excitation frequencies. A higher frequency has the potential of increased dissipation by viscous damping.

2 System dynamics

2.1 Main system

The studied main system, a linear lumped MDOF system, is governed by the following dynamic equation:

$$M\ddot{x}(t) + C\dot{x}(t) + Kx(t) = F(t) \tag{1}$$

with $x(t) = \left[x_1(t)\ x_2(t)\ \dots\ x_n(t)\right]^T \in \mathbb{R}^n$ the displacement vector of each DOF, $M \in \mathbb{R}^{n\times n}$ the mass matrix, $C \in \mathbb{R}^{n\times n}$ the viscous damping matrix, $K \in \mathbb{R}^{n\times n}$ the stiffness matrix and $F \in \mathbb{R}^n$ the force vector. Assuming proportional damping ($C = aM + bK$, $a, b \in \mathbb{R}_+$), the n quadratic eigenvalues ω_i^2 with corresponding eigenvectors are found as:

$$\begin{cases} det(K - M\omega^2) = 0 \\ (K - M\omega_i^2)e_i = 0 \end{cases} \tag{2}$$

Let $E = [e_1 \quad e_2 \quad \dots \quad e_n] \quad \in \mathbb{R}^{n\times n}$ be the eigenvector matrix, then physical coordinates can be transformed into the modal coordinates $p \in \mathbb{R}^n$ as $x(t) = Ep(t)$:

$$M_p\ddot{p}(t) + C_p\dot{p}(t) + K_p p(t) = E^T F(t) \tag{3}$$

with $M_p = E'ME$ the diagonal modal mass matrix, $C_p = E'CE$ the diagonal modal damping matrix and $K_p = E'KE$ the diagonal modal stiffness matrix. The main system is effectively transformed in n decoupled linear oscillators with natural frequencies ω_i

A vibration absorber with an uneven power series stiffness is added on lumped coordinate m. The new compound system, depicted on fig.1 is described by:

$$\begin{cases} M\ddot{x} + C\dot{x} + Kx + m_{na}\delta_{i,m}\ddot{x}_{na} = F \\ m_{na}\ddot{x}_{na} + c_{na}(\dot{x}_{na} - \dot{x}_m) + k_{lin}(x_{na} - x_m) \\ \qquad + \sum_{\ell=1}^{L} k_{nl,2\ell+1}(x_{na} - x_m)^{2\ell+1} = 0 \end{cases} \tag{4}$$

with $\delta_{i,m} = [0\ 0\ ...\ 0\ 1\ 0\ ...0]^T \in \mathbb{R}^n$. If the mass, damping and stiffness values of the absorber are considerably smaller compared to those of the main structure, the compound system can still be accurately described by the modal decomposition $x = Ep$:

$$\begin{cases} M_p\ddot{p} + C_p\dot{p} + K_p p + m_{na}e_{m,*}^T\ddot{x}_{na} = E^T F \\ m_{na}\ddot{x}_{na} + c_{na}(\dot{x}_{na} - \dot{x}_m) + k_{lin}(x_{na} - x_m) \\ \qquad + \sum_{\ell=1}^{L} k_{nl,2\ell+1}(x_{na} - x_m)^{2\ell+1} = 0 \end{cases} \tag{5}$$

with $e_{m,*}$ denoting the m-th row of E, so the m-th component of each eigenvector.

Further analysis on the absorber assumes that the main system imposes a ground motion to the absorber, while the absorber has no effect on the main system. This method was first proposed in [9]. If the absorber has no damping, and has small mass and stiffness (low coupling) compared to the main system, this assumption holds. The ground motion of the absorber is then the m-th row of $x = Ep$. This will reduce the problem to a first-order nonlinear oscillator, of which the cases $L = 1$ and $L = 2$ are well established for small nonlinearities [7, 8].

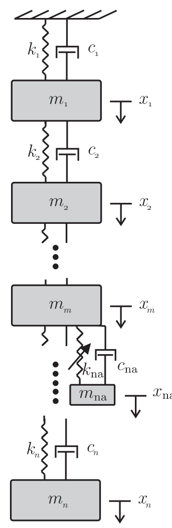

Figure 1: The studied compound system, a lumped multivariable linear system, with a nonlinear absorber on physical coordinate m. The spring k_{na} lumps both the linear and nonlinear part of the stiffness.

2.2 Imposed motion

If interaction between absorber and main system is neglected, and no forces are exerted on the main system ($F = 0$), the attachment point of the absorber, coordinate m, has displacement $x_m \triangleq z$:

$$z(t) = \sum_{i=1}^{N} e_i(m) \left(p_i(0)cos(\omega_i t) + \frac{\dot{p}_i(0)}{\omega_i} sin(\omega_i t) \right) \tag{6}$$

This motion is seen as the imposed ground motion on the absorber's stiffness, see fig.2. By introducing the relative absorber displacement $x_r = x_{na} - z$, the undamped absorber dynamics becomes:

$$\ddot{x_r} + \omega_0^2 x_r + \sum_{\ell=1}^{L} \gamma_{nl,2\ell+1} x^{2\ell+1} = \sum_{i=1}^{N} X_i sin(\omega_i t + \phi_i) \tag{7}$$

with $\omega_0^2 = k_{lin}/m_{na}$, $\gamma_{nl,2\ell+1} = k_{nl,2\ell+1}/m_{na}$, $X_i = e_i(m)\sqrt{\omega_i^4 p_i(0)^2 + \omega_i^2 \dot{p}_i(0)^2}$,
and $\phi_k = arctan(\omega_k p_k(0)/\dot{p_0}) + \pi$.

The multi-dimensional nonlinear dynamical problem is effectively reduced to a one dimensional nonlinear problem, greatly diminished complexity. In the next section, the conditions for combination resonance on this simplified model is analysed.

3 Analysis simplified absorber model

3.1 Definition

The excited oscillator, with a uneven power series stiffness, has the following dynamics:

$$\ddot{x} + c\dot{x} + \omega_0^2 x + \sum_{\ell=1}^{L} \gamma_{nl,2\ell+1} x^{2\ell+1} = F_{nl}(t) \tag{8}$$

with c the damping and $F_{nl}(t)$ the excitations on the nonlinear absorber. Nayfeh [7, 7] showed for $L = 1$ (the duffing oscillator) and small damping, the response of the oscillator consists of a linear term (as if $\gamma_{nl,3} = 0$), and under certain conditions, combination resonances. These resonances add a steady state term to the response with frequency $\omega_0 = \sqrt{k_{lin}/m_{na}}$, the natural frequency if the nonlinear part is neglected. From here on, the response term with this frequency is called the *combination response*. This behaviour does not occur in linear oscillators, where the term with ω_0 is called the free response always, and decays to zero. To emphasize the small nonlinear behaviour, (8) is rewritten as:

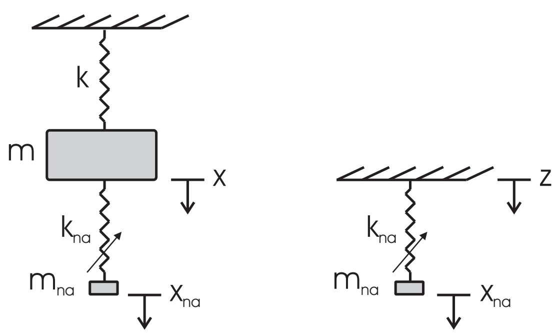

Figure 2: The absorber is decoupled from the main system and the motion of the main system is seen as the imposed motion on the absorber's stiffness

$$\ddot{x}+2\varepsilon\mu\dot{x}+\omega_0^2 x+\varepsilon\sum_{\ell=1}^{L}\alpha_{2\ell+1}x^{2\ell+1}=F_{nl}(t) \tag{9}$$

with $2\varepsilon\mu=c$, $\varepsilon\alpha_{2\ell+1}=\gamma_{nl,2\ell+1}$ and ε a small parameter, $\mathscr{O}(\varepsilon)\ll 1$.

The excitation is periodic, possibly containing multiple frequencies (N) that do not necessarily commute:

$$F_{nl}(t)=\sum_{i=1}^{N}K_i cos(\omega_i t+\theta_i) \tag{10}$$

The following vectors are introduced

$$\begin{aligned}
&\vec{\Omega}=[\omega_1 \quad \omega_2 \quad \dots \quad \omega_N \quad -\omega_1 \quad \dots \quad -\omega_N]\\
&\vec{\Theta}=[\theta_1 \quad \theta_2 \quad \dots \quad \theta_N \quad -\theta_1 \quad \dots \quad -\theta_N]\\
&\vec{B}_{2\ell+1} \quad \ell\in\{1,2,\dots,L\}
\end{aligned} \tag{11}$$

with $\vec{B}_{2\ell+1}\in\mathbb{R}^{2N}$, $\{\vec{B}_{2\ell+1}\}_i\in\mathbb{N}_0$ and $sum\{\{\vec{B}_{2\ell+1}\}_i\}=2\ell+1$. It can be shown that for any ℓ, if $\vec{B}_{2\ell+1}\vec{\Omega}^T=\omega_0+\varepsilon\sigma$, the combination term persists in an interval of σ, with σ a frequency shift parameter.

3.2 Combination Frequency Response function

In this subsection, a relation is sought between the frequency shift parameter σ and the combination response for $L=1$, the duffing oscillator. Two cases are analysed, the case $N=2$ with $\vec{B}_3=[2 \quad 1 \quad 0 \quad 0]$, or combination $2\omega_1+\omega_2=\omega_0+\varepsilon\sigma$ and the case $N=3$ with $\vec{B}_3=[1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0]$, or combination $\omega_1+\omega_2+\omega_3=\omega_0+\varepsilon\sigma$.

The displacement $x(t)$ is expanded in a power series in ε up to the first order. Additionally, it is assumed that the response is expressed in two independent time scales; the fast $T_0=t$ and slow $T_1=\varepsilon t$ scale [7]:

$$x(t,\varepsilon)=x_0(T_0,T_1)+\varepsilon x_1(T_0,T_1) \tag{12}$$

The displacement in then seen as mainly consisting of x_0, with x_1 a correction term, as ε is small. For this assumption to hold, x_1 will be kept at the same order as x_0 by setting secular terms of x_1 to zero. With the two independent time scales, the 'regular' time derivative becomes:

$$\frac{d}{dt}=\frac{\partial}{\partial T_0}+\varepsilon\frac{\partial}{\partial T_1} \tag{13}$$

With (12) and (13), the oscillator dynamics (9) are rewritten in two differential equations, by collecting the terms in ε^0 and ε^1:

$$\begin{aligned}
&\frac{\partial^2 x_0}{\partial T_0^2}+\omega_0^2 x_0=\sum_{i=1}^{N}K_i cos(\omega_i T_0+\theta_i)\\
&\frac{\partial^2 x_1}{\partial T_0^2}+\omega_0^2 x_1=-2\frac{\partial^2 x_0}{\partial T_0\partial T_1}-2\mu\frac{\partial x_0}{\partial T_0}-\alpha_3 x_0^3
\end{aligned} \tag{14}$$

Solving the first differential equation of (14) gives an expression for x_0:

$$x_0=A(T_1)e^{j\omega_0 t}+\sum_{i=1}^{N}0.5\frac{K_i}{\omega_0^2-\omega_i^2}e^{j(\omega_i t+\theta_i)}+cc \tag{15}$$

with *cc* denoting the complex conjugate. Note that if one of the excitation frequencies equals the natural frequency, the first equation of (14) will result in an infinity large response. This is not considered here, the goal is to achieve combination resonance in an absorber. Substituting (15) in the second equation of (14) and eliminating secular terms, the terms with frequency ω_0, gives:

$$2j\omega_0(\frac{\partial A}{\partial T_1}+\mu A)+\omega_0\alpha_3\Gamma_2 A+3\alpha_3 A^2\bar{A}+\omega_0\alpha_3\Gamma_1 e^{j(\sigma T_1+\vec{B}_3\vec{\Theta}^T)} \tag{16}$$

with Γ_1 and Γ_2 for the $N=2$ and $\vec{B}_3=[2\quad 1\quad 0\quad 0]$ case:

$$\begin{aligned}\Gamma_1 &= \frac{3}{8\omega_0}\frac{K_1^2}{(\omega_0^2-\Omega_n^2)^2}\frac{K_2}{\omega_0^2-\Omega_2^2}\\ \Gamma_2 &= \frac{3}{4\omega_0}(\frac{K_1}{\omega_0^2-\Omega_n^2}+\frac{K_2}{\omega_0^2-\Omega_2^2})\end{aligned} \tag{17}$$

and for the case of $N=3$ and $\vec{B}_3=[1\quad 1\quad 1\quad 0\quad 0\quad 0]$:

$$\begin{aligned}\Gamma_1 &= \frac{3}{8\omega_0}\frac{K_1}{\omega_0^2-\Omega_n^2}\frac{K_2}{\omega_0^2-\Omega_2^2}\frac{K_3}{\omega_0^2-\Omega_3^2}\\ \Gamma_2 &= \frac{3}{4\omega_0}(\frac{K_1}{\omega_0^2-\Omega_n^2}+\frac{K_2}{\omega_0^2-\Omega_2^2}+\frac{K_3}{\omega_0^2-\Omega_3^2})\end{aligned} \tag{18}$$

Let $A(T_1)=\frac{1}{2}ae^{j\beta}$, replace in (16) and separate real and imaginary parts:

$$\begin{aligned}\frac{\partial a}{\partial T1} &= -\mu a-\alpha_3\Gamma_1 sin(\sigma T_1-\beta+\vec{B}_3\vec{\Theta}^T)\\ a\frac{\partial \beta}{\partial T1} &= \alpha_3\Gamma_2 a+\frac{3\alpha_3}{8\omega_0}a^3+\alpha_3\Gamma_1 cos(\sigma T_1-\beta+\vec{B}_3\vec{\Theta}^T)\end{aligned} \tag{19}$$

with $\delta=\sigma T_1-\beta+\vec{B}_3\vec{\Theta}^T$, the differential equation is made autonomous:

$$\begin{aligned}\frac{\partial a}{\partial T1} &= -\mu a-\alpha_3\Gamma_1 sin(\delta)\\ a\frac{\partial \delta}{\partial T1} &= (\sigma-\alpha_3\Gamma_2)a-\frac{3\alpha_3}{8\omega_0}a^3-\alpha_3\Gamma_1 cos(\delta)\end{aligned} \tag{20}$$

When considering (20) in steady state, a relation between the frequency shift parameter σ and combination resonance amplitude a is found:

$$\left[\mu^2+\left(\sigma-\alpha_3\Gamma_2-\frac{3\alpha_3}{8\omega_0}a^2\right)^2\right]a^2=\alpha_3^2\Gamma_1^2 \tag{21}$$

or:

$$\sigma=\alpha_3\Gamma_2+\frac{3\alpha_3}{8\omega_0}a^2\pm\sqrt{\frac{\alpha_3^2\Gamma_1}{a^2}-\mu^2} \tag{22}$$

From here on this relation will called the combination frequency response function (FRF). An numerical example of a combination FRF for $N=2$ is shown on fig. 3. It is different to a normal FRF, where the $x-axis$ is the excitation frequency and tuning is simple. In the combination FRF, there a several ways to change the $x-axis$ parameters σ:

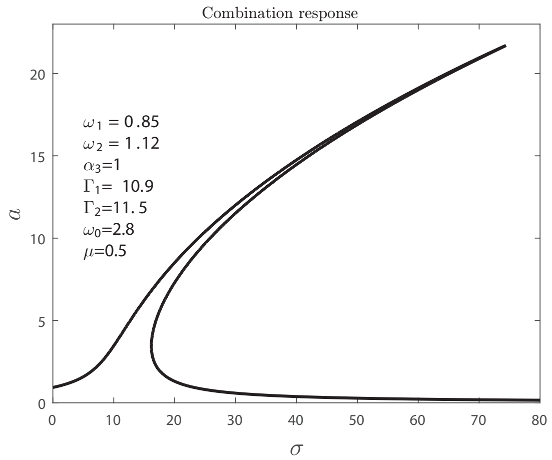

Figure 3: Amplitude of steady state free response in function of the frequency shift

- Change ε while keeping ω_0 and ω_i's constant, changing c and γ_3 in (9)
- Change ω_0 while keeping ω_i's and ε constant, changing Γ_i's and the shape of (22)
- Change ω_i while keeping ω_0 and ε constant, changing Γ_i's and the shape of (22)

To tune the combination response later on, no frequencies will actually be changed, so the first method is used. This way the shape of the proposed combination FRF does not change.

As seen on fig. 3, there is a range of σ's with three solutions in a. In that case, the middle value of a is an unstable steady state. Which of the other two stable solutions is reached depends on the initial conditions [8].

3.3 Energy threshold

It is shown here that the excitations amplitudes K_i need to be above a threshold, for combination resonance to occur. As the oscillator is used as an absorber, the excitations are actually vibrating modes of a main system. The magnitude of these modes depend on the initial conditions, or initial energy in the main system. Therefore, this threshold is some kind of energy threshold. Also, the absorber will absorber energy, reducing the vibrations in the main system and consequently reducing the K_i coefficients.

To visualize the threshold, consider the $N = 2$ case and let $K_1 = K_2 = K$. By keeping σ, ω_0,ω_i's, α_3 and μ constant, (22) implicitly plots a and K, as K appears in the Γ_i's. A numerical example of combination response amplitude in function of K is plotted on 4.

A clear threshold is seen, K should be above 15.8 for a sufficient combination response. Thus, the modes participating in the combination resonance should sufficiently present in the main system at the absorber position so that the absorber is well beyond this threshold. Eventually, dissipation will reduce one the modes of vibration below this threshold value, significantly decreasing the combination response. However, going beyond threshold to much also greatly reduces a, so a too hard vibrating main system will not induce combination resonance in the absorber.

To lower the threshold, a structural *modification* is proposed which increases the K_i coefficients in (10), without changing the main systems displacements (so constant X_i's). A negative stiffness $-k_g$ is added to the absorber and connected to the ground of the main system, see fig. 5.

If the linear stiffness from the absorber is increased with the same k_g, the structural modification changes the dynamics of the decoupled absorber (9) to:

$$\ddot{x}_r + \omega_0^2 x_r - \kappa z + \varepsilon \sum_{\ell=1}^{L} \alpha_{2\ell+1} x^{2\ell+1} = \sum_{i=1}^{N} X_i sin(\omega_i t + \phi_k) \tag{23}$$

with $\kappa = k_g/m_{na}$. By considering a lightly damped main system, $\ddot{p}_i = -\omega_i^2 p_i$, this can be rewritten as:

$$\begin{aligned} \ddot{x}_r + \omega_0^2 x_r + \varepsilon \sum_{\ell=1}^{L} a_{2\ell+1} x^{2\ell+1} = \\ \sum_{i=1}^{N} X_i (1 + \frac{\kappa}{\omega_i^2}) sin(\omega_i t + \phi_k) \end{aligned} \tag{24}$$

The excitation terms K_i are increased by a factor $(1 + \frac{\kappa}{\omega_i^2})$, decreasing the threshold value for the same X_i's. However, the negative stiffness should not be too large, as too large K's decreases the value of a.

4 Simulations

To demonstrate combination resonances in a weakly nonlinear absorber, a two DOF main system is considered, with parameters found on tab. 1. The absorber is attached to the second DOF. This system has eigenfrequencies 0.89 rad/s and 1.12 rad/s and is excited with an initial speed of 10 m/s on the ground floor. To tune the absorber, ω_0, α and $\mu = 0$ are proposed. The ω_0 fixes the product $\sigma\varepsilon$. By considering the imposed motion simplification, the modes of the main system are seen as excitations, giving values for Γ_1

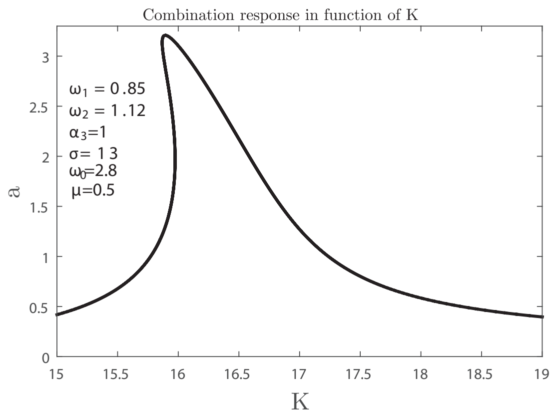

Figure 4: Amplitude of combination response in function of the frequency shift. If $K(0) > 15.8$, a sudden decrease of a is seen as damping reduces K below 15.8, the threshold value.

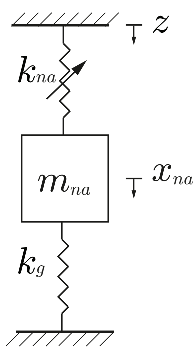

Figure 5: The absorber is modified by adding a negative stiffness from absorber to ground.

DOF	1	2
m_i	$10\,kg$	$0.5\,kg$
k_i	$10\,N/m$	$0.5\,N/m$

Table 1: Main system with 2 DOF

Parameter	ω_0	α	μ	Γ_1	Γ_2	κ
	2.86	1	0.5	6.23	7.56	0.2

Table 2: Absorber parameters

and Γ_2. Now that every parameter except ε and σ is known, the combination FRF (21) can be constructed. Tuning σ results in an ε and an a. The ε in its turn determines the physical parameter of the $2\mu\varepsilon$ and $\varepsilon\alpha_3$. Beware that the tuning is different for different initial conditions of the main systems, as this influences $\Gamma 1$ and $\Gamma 2$. The chosen parameters for the tuning of the absorber are summarized in tab.2.

Although a damping is chosen as to dissipate energy, it has little effect on the combination FRF's shape around the bifurcation value of σ [7, 8], which is the region of interest.

The combination FRF, together with the chosen σ's (10 and 14) are depicted on fig. 6. The first σ will result in a large steady state response, while the second σ will result either in a small or large steady state. No attempt was made to put the free response in the larger of the two steady states, as this requires different initial conditions of the main system, altering Γ_1, Γ_2, resulting in a new combination FRF shape. The absorber is then not purely tunable with the parameter σ, increasing tuning difficulty, so the largest steady state is not considered here.
In a first simulation, the absorber is tuned to $\sigma = 10$. The initial response is plotted on fig. 7. From the time series and the wavelet transform, it is clear that the relative absorber movement x_r contains both the 2 modes of the main system and the combination response term of frequency ω_0. The displacement of the first floor only contains the 2 modes of the linear main structure. As a comparison, the wavelet transform of the same absorber without nonlinear terms is shown as well, having no component with frequency ω_0. Over time, the energy is dissipated in the absorber, reducing the main system's displacement. As predicted in fig. 4, this will cause a bifurcation where the combination response term diminishes greatly. This sudden decrease in amplitude happens at $t = 4800$ in the simulation (fig. 8). After the bifurcation, the nonlinear absorber works as well as its linear counterpart. This simulation thus shows the existence of combination resonance and the energy threshold. It also highlights the limited possibilities of combination resonances in weakly nonlinear absorbers, as the energy dissipation is very slow. This is caused by the assumption of small absorber damping $2\varepsilon\mu$. The potential of the increased frequency for faster dissipation is thus not used fully.
For $\sigma = 14$, the absorber's free response goes to the lower steady state value, as only a slight component of ω_0 is present (fig. 9)

5 Conclusion

This study has investigated if combination resonance is possible in a slightly nonlinear vibration absorber. By reducing the problem to a SDOF nonlinear oscillator with the imposed motion method, classic nonlinear techniques were applied, which led to the so called combination FRF. It was found that the main system's vibrations should be both no too high or low for combination resonance to occur in the added absorber. Even when the vibrations are within good range, energy dissipation will eventually cause a sudden decrease of the combination response. To delay this bifurcation, a structural modification was proposed by connecting a negative stiffness from absorber to the main system's ground. It was shown that it is possible to tune the small damping and nonlinearity of the absorber to ensure combination resonance. To verify the tuning method, an absorber was added to a 2DOF linear main system. This system underwent a ground impulse,

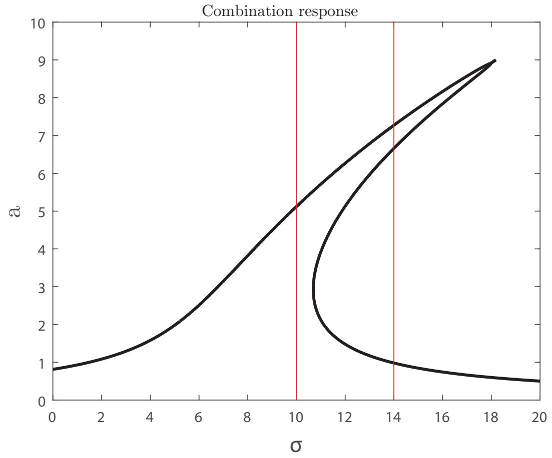

Figure 6: Amplitude of steady state free response in function of the frequency shift for the absorber used in 2DOF system. The considered σ's are 10 and 14.

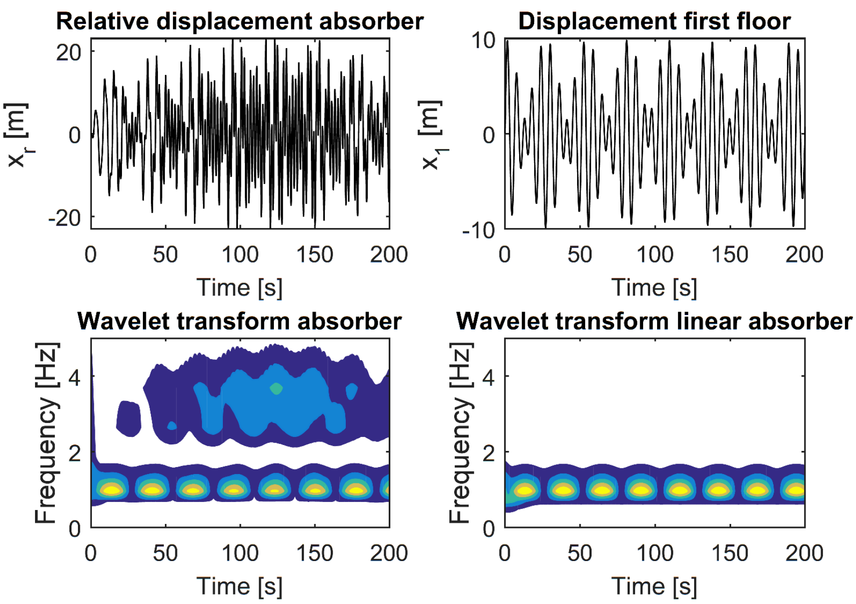

Figure 7: $\sigma = 10$, time series and Wavelet transform of relative absorber movement (Left). In both the time and frequency domain it is clear that besides the two modes a higher frequency component is present. The first floor has two modes (Upper right). The wavelet transform of a time series of the absorber without nonlinear part only has the two modes.

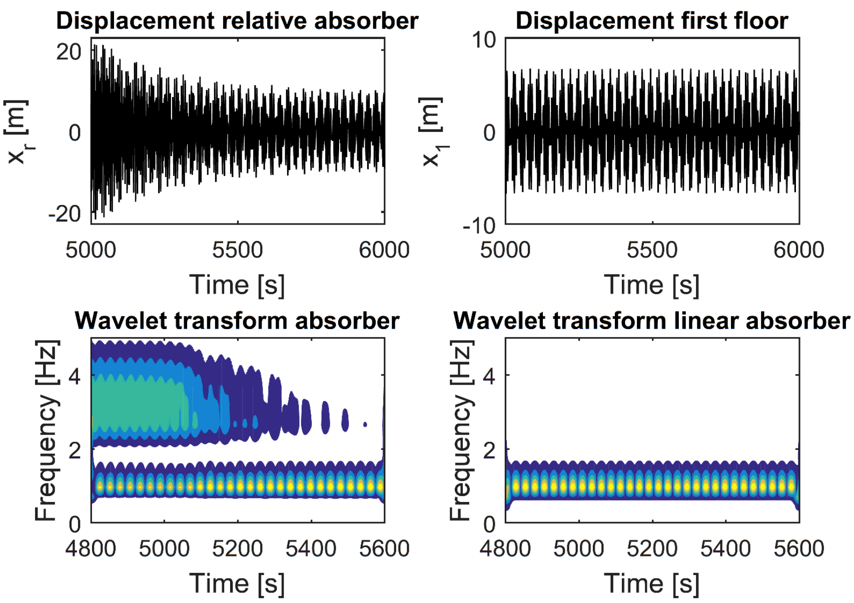

Figure 8: $\sigma = 10$,A large reduction in relative absorber movement, caused by the bifurcation. The free response mode dissapears, as is clear in the wavelet transform (Left). The first floor displacement has diminished about 40 (Upper right).

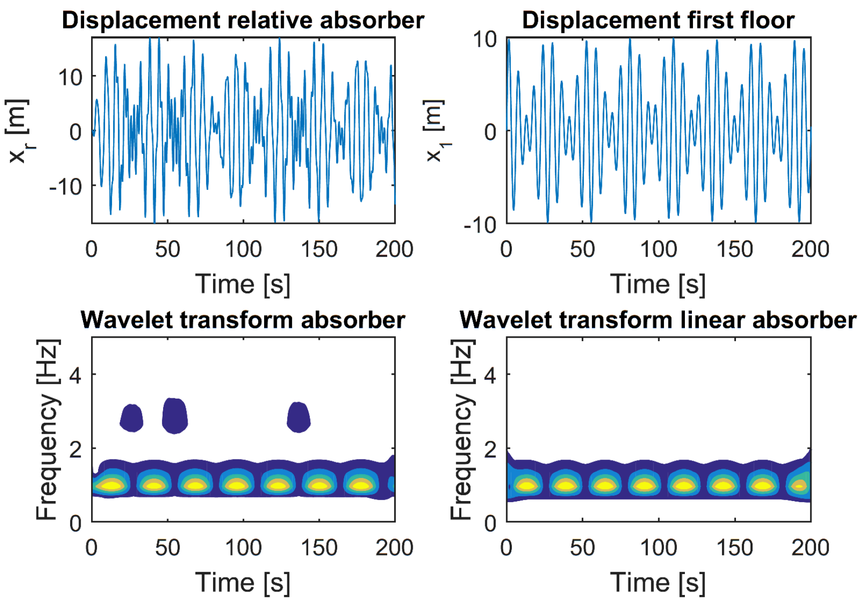

Figure 9: $\sigma = 14$, besides a small transient free response, the absorber does not exhibits combination resonance, it works as good as the linear absorber.

causing the tuned absorber exhibit combination resonance. After some time, the main system's vibration were too low, after which the combination resonance disappeared. The small damping and nonlinearity caused a slow dissipation of the frequencies. In the future, the possibilities of combination resonances in absorber with stronger damping and nonlinearities will be investigated to make better use of the potential of increased dissipation.

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